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Surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number | | | | | |

Candidate Number | | | | |

Further Pure Mathematics F1

Advanced/Advanced Subsidiary

Monday 14 May 2018 – Afternoon
Time: 1 hour 30 minutes

Paper Reference
WFM01/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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1. Use the standard results for $\sum_{r=1}^{n} r$ and for $\sum_{r=1}^{n} r^2$ to show that, for all positive integers n,

$$\sum_{r=1}^{n} r(r+3) = \frac{n}{a}(n+1)(n+b)$$

where a and b are integers to be found.

(4)

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Question 1 continued

Q1

(Total 4 marks)

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2. The transformation represented by the 2×2 matrix **P** is an anticlockwise rotation about the origin through 45 degrees.

(a) Write down the matrix **P**, giving the exact numerical value of each element.

(1)

$$\mathbf{Q} = \begin{pmatrix} k\sqrt{2} & 0 \\ 0 & k\sqrt{2} \end{pmatrix}, \text{ where } k \text{ is a constant and } k > 0$$

(b) Describe fully the single geometrical transformation represented by the matrix **Q**.

(2)

The combined transformation represented by the matrix **PQ** transforms the rhombus R_1 onto the rhombus R_2.

The area of the rhombus R_1 is 6 and the area of the rhombus R_2 is 147

(c) Find the value of the constant k.

(4)

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Question 2 continued

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Question 2 continued

Q2

(Total 7 marks)

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3.

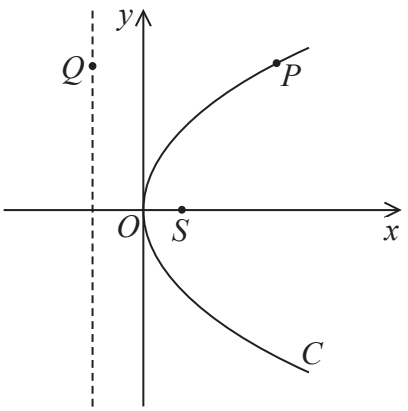

Figure 1

Figure 1 shows the parabola C which has cartesian equation $y^2 = 6x$. The point S is the focus of C.

(a) Find the coordinates of the point S.

(1)

The point P lies on the parabola C, and the point Q lies on the directrix of C. PQ is parallel to the x-axis with distance $PQ = 14$

(b) State the distance SP.

(1)

Given that the point P is above the x-axis,

(c) find the exact coordinates of P.

(3)

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Question 3 continued

Q3

(Total 5 marks)

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4.

$$\mathbf{A} = \begin{pmatrix} 2p & 3q \\ 3p & 5q \end{pmatrix}$$

where p and q are non-zero real constants.

(a) Find $\mathbf{A}^{-1}$ in terms of p and q.

(3)

Given $\mathbf{XA} = \mathbf{B}$, where

$$\mathbf{B} = \begin{pmatrix} p & q \\ 6p & 11q \\ 5p & 8q \end{pmatrix}$$

(b) find the matrix $\mathbf{X}$, giving your answer in its simplest form.

(4)

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Question 4 continued

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Question 4 continued

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Question 4 continued

Q4

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5. Given that

$$z^4 - 6z^3 + 34z^2 - 54z + 225 \equiv (z^2 + 9)(z^2 + az + b)$$

where a and b are real numbers,

(a) find the value of a and the value of b.

(2)

(b) Hence find the exact roots of the equation

$$z^4 - 6z^3 + 34z^2 - 54z + 225 = 0$$

(4)

(c) Show your roots on a single Argand diagram.

(2)

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Question 5 continued

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Question 5 continued

Q5

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6.

$$f(x) = \frac{2(x^3 + 3)}{\sqrt{x}} - 9, \quad x > 0$$

The equation $f(x) = 0$ has two real roots α and β, where $0.4 < \alpha < 0.5$ and $1.2 < \beta < 1.3$

(a) Taking 0.45 as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to find a second approximation to α, giving your answer to 3 decimal places.

(5)

(b) Use linear interpolation once on the interval [1.2, 1.3] to find an approximation to β, giving your answer to 3 decimal places.

(4)

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Question 6 continued

Q6

(Total 9 marks)

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7. It is given that α and β are roots of the equation $5x^2 - 4x + 3 = 0$

Without solving the quadratic equation,

(a) find the exact value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$

(5)

(b) find a quadratic equation which has roots $\frac{3}{\alpha^2}$ and $\frac{3}{\beta^2}$

giving your answer in the form $ax^2 + bx + c = 0$, where a, b and c are integers to be found.

(4)

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Question 7 continued

Q7

(Total 9 marks)

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8. Prove by induction that, for $n \in \mathbb{Z}^+$

$$\begin{pmatrix} a & 0 \\ 1 & b \end{pmatrix}^n = \begin{pmatrix} a^n & 0 \\ \dfrac{a^n - b^n}{a - b} & b^n \end{pmatrix}$$

where a and b are constants and $a \neq b$.

(5)

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Question 8 continued

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Question 8 continued

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Question 8 continued

Q8

(Total 5 marks)

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9. Given that

$$\frac{z - k\mathrm{i}}{z + 3\mathrm{i}} = \mathrm{i}\,, \text{ where } k \text{ is a positive real constant}$$

(a) show that $z = -\frac{(k+3)}{2} + \frac{(k-3)}{2}\mathrm{i}$

(4)

(b) Using the printed answer in part (a),

(i) find an exact simplified value for the modulus of z when $k = 4$

(ii) find the argument of z when $k = 1$. Give your answer in radians to 3 decimal places, where $-\pi < \arg z < \pi$

(4)

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Question 9 continued

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Question 9 continued

Q9

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10. The rectangular hyperbola H has equation $xy = 144$. The point P, on H, has coordinates $\left(12p, \frac{12}{p}\right)$, where p is a non-zero constant.

(a) Show, by using calculus, that the normal to H at the point P has equation

$$y = p^2x + \frac{12}{p} - 12p^3$$

(5)

Given that the normal through P crosses the positive x-axis at the point Q and the negative y-axis at the point R,

(b) find the coordinates of Q and the coordinates of R, giving your answers in terms of p.

(3)

(c) Given also that the area of triangle OQR is 512, find the possible values of p.

(5)

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Question 10 continued

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Question 10 continued

Q10

(Total 13 marks)

TOTAL FOR PAPER: 75 MARKS

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