Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number | Candidate Number

Wednesday 6 November 2019

Morning (Time: 2 hours 30 minutes) | Paper Reference **WMA02/01**

Mathematics

International Advanced Level
Core Mathematics C34

You must have:
Mathematical Formulae and Statistical Tables (Blue), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 14 questions in this question paper. The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

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1. (a) Express $3\sin x - \cos x$ in the form $R\sin(x - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give the exact value of R and give the value of α, in radians, to 3 decimal places.

(3)

The temperature, θ°C, inside a building on a particular day, is modelled by the equation

$$\theta = 19 + 3\sin\left(\frac{\pi t}{12} + 4\right) - \cos\left(\frac{\pi t}{12} + 4\right), \qquad 0 \leqslant t < 24$$

where t is the number of hours after midnight.

(b) Using the answer to part (a),

(i) state the minimum value of θ predicted by this model,

(ii) find the value of t, to 2 decimal places, when this minimum occurs.

(3)

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Question 1 continued

Q1

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2. $$f(x) = \left(\frac{1}{3} - x\right)^{-2} \qquad |x| < \frac{1}{3}$$

(a) Find the binomial expansion of $f(x)$, in ascending powers of x, up to and including the term in x^3, giving each coefficient in its simplest form.

(4)

$$g(x) = \left(\frac{1}{3} - x\right)^{-2}(a + bx) \qquad |x| < \frac{1}{3}$$

where a and b are constants.

Given that, in the series expansion of $g(x)$, the coefficient of x is 3 and the coefficient of x^2 is 27

(b) find the value of a and the value of b.

(3)

(c) Hence find the coefficient of x^3 in the series expansion of $g(x)$.

(2)

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Question 2 continued

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Question 2 continued

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Question 2 continued

Q2

(Total 9 marks)

3.

$$f(x) = \frac{5x + 2}{x - 3} \qquad x \in \mathbb{R}, \quad x \neq 3$$

$$g(x) = 2x^2 - 1 \qquad x \in \mathbb{R}$$

(a) Write down the range of g.

(1)

(b) Find fg(x), simplifying your answer.

(2)

(c) Find $f^{-1}(x)$.

(3)

(d) Find the exact values of x for which

$$f^{-1}(x) = f(x)$$

giving your answers as fully simplified surds.

(4)

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Question 3 continued

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Question 3 continued

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Question 3 continued

Q3

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4. The curve C has equation

$$y = x\cos 2x \qquad 0 \leqslant x \leqslant \frac{\pi}{4}$$

The curve has a turning point at the point P.

(a) Show, using calculus, that the x coordinate of P is a solution of the equation

$$x = \frac{1}{2}\arctan\left(\frac{1}{2x}\right)$$

(4)

(b) Starting with $x_0 = 0.5$ use

$$x_{n+1} = \frac{1}{2}\arctan\left(\frac{1}{2x_n}\right)$$

to calculate the value of x_1 and the value of x_2, giving your answers to 4 decimal places.

(3)

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Question 4 continued

Q4

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5. The height, h metres, of a shrub, t years after it was planted, is modelled by the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{2h^{\frac{3}{2}}}{5t^2} \qquad t > 0$$

(a) Given that $h = 1$ when $t = 1$, show that

$$h = \frac{at^2}{(1 + bt)^2}$$

where a and b are constants to be found.

(7)

(b) Hence find, according to the model, the limit of the height of the shrub.

(2)

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Question 5 continued

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Question 5 continued

Q5

(Total 9 marks)

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6. (a) Prove that

$$\frac{\sec x}{1+\sec x} - \frac{\sec x}{1-\sec x} \equiv 2\operatorname{cosec}^2 x \qquad x \neq n\pi, \quad n \in \mathbb{Z}$$

(3)

(b) Hence solve, for $0 < \theta < \pi$

$$\frac{\sec 2\theta}{1+\sec 2\theta} - \frac{\sec 2\theta}{1-\sec 2\theta} = 3 - 2\cot^2 2\theta$$

giving your answers in radians to 3 significant figures.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(6)

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Question 6 continued

Question 6 continued

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Question 6 continued

Q6

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7. Given that

$$\frac{2x^2 - 3}{(3-2x)(1-x)^2} \equiv \frac{A}{3-2x} + \frac{B}{1-x} + \frac{C}{(1-x)^2}$$

(a) find the values of the constants A, B and C.

(4)

(b) Hence find

$$\int \frac{2x^2 - 3}{(3-2x)(1-x)^2}\,\mathrm{d}x$$

(4)

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Question 7 continued

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Question 7 continued

Q7

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8. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = \begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 2 \\ -3 \end{pmatrix} \qquad l_2 : \mathbf{r} = \begin{pmatrix} 9 \\ -7 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -5 \\ 2 \end{pmatrix}$$

where λ and μ are scalar parameters.

(a) Show that l_1 and l_2 meet, and find the position vector of their point of intersection.
(6)

The point P with position vector $\begin{pmatrix} 11 \\ p \\ -6 \end{pmatrix}$, where p is a constant, lies on l_1

(b) Find the value of p.
(1)

Given that point Q lies on l_2 such that PQ is perpendicular to l_2

(c) find the exact coordinates of the point Q.
(4)

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Question 8 continued

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Question 8 continued

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Q8

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9. Given that a is a positive constant,

(a) on separate diagrams, sketch the graph with equation

(i) $y = a - |x|$

(ii) $y = |3x - 2a|$

Show on each sketch the coordinates, in terms of a, of each point at which the graph crosses or meets the axes.

(4)

(b) Find, in terms of a, the values of x for which

$$a - |x| = |3x - 2a|$$

(4)

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Question 9 continued

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Question 9 continued

Q9

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10. (a) Using the substitution $u = 2x - 1$, show that

$$\int_2^5 \frac{(3x+2)^2}{2x-1}\,\mathrm{d}x = 72 + \frac{49}{8}\ln 3$$

(6)

The curve C has equation

$$y = \frac{3x+2}{2\sqrt{2x-1}} \qquad x > 1$$

The finite region R is bounded by C, the x-axis and the lines with equations $x = 2$ and $x = 5$

The region R is rotated through 2π radians about the x-axis to form a solid of revolution.

(b) Using the result from part (a), find the exact value of the volume of the solid generated.

(2)

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Question 10 continued

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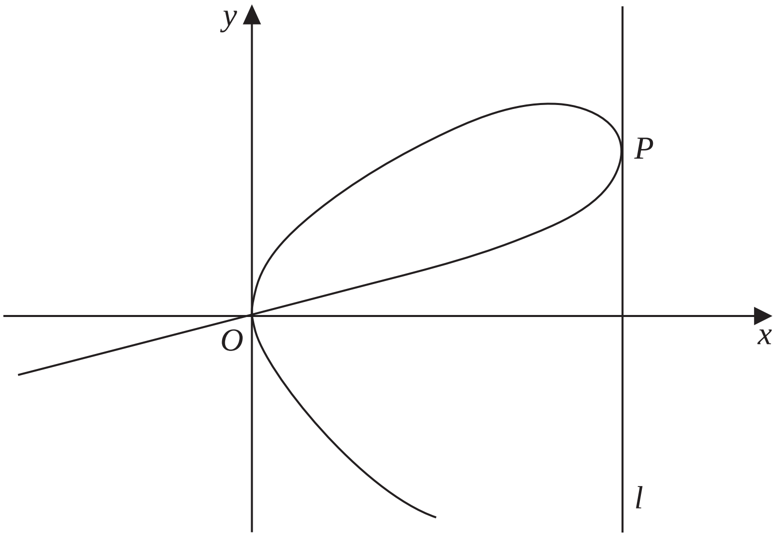

Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$2x^2 + y^3 = kxy$$

where k is a positive constant.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x, y and k.

(4)

The line l is parallel to the y-axis and touches the curve at the point P, as shown in Figure 1.

(b) Find, in terms of k, the coordinates of the point P.

(5)

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Question 11 continued

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Question 11 continued

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Question 11 continued

(Total 9 marks)

Q11

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12. A scientist is studying a population of fish in a lake. The number of fish, N, in the population, t years after the start of the study, is modelled by the equation

$$N = \frac{250\mathrm{e}^{0.2t}}{1 + 0.25\mathrm{e}^{0.2t}} \qquad t \geqslant 0$$

(a) Find, according to the model, the number of fish in the lake at the start of the study.
(1)

(b) Find, according to the model, the value of t when there are 800 fish in the lake, giving your answer to the nearest integer.
(3)

(c) Show that

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{50\mathrm{e}^{0.2t}}{\left(1 + 0.25\mathrm{e}^{0.2t}\right)^2}$$

(2)

Given that $t = T$ when $\dfrac{\mathrm{d}N}{\mathrm{d}t} = 10$

(d) find the value of T to one decimal place.
(4)

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13.

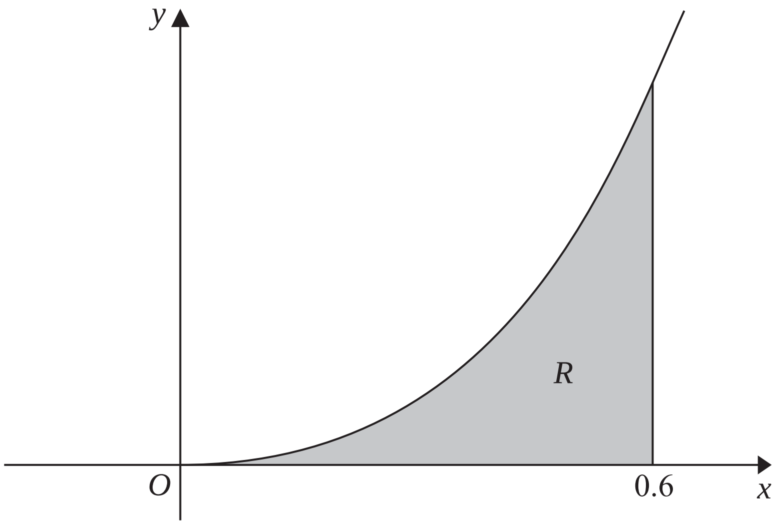

Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = x^2 4^x$$

The finite region R, shown shaded in Figure 2, is bounded by the curve C, the x-axis and the line with equation $x = 0.6$

The table below shows corresponding values of x and y for $y = x^2 4^x$

x	0	0.1	0.2	0.3	0.4	0.5	0.6
y	0	0.0115	0.0528		0.2786	0.5	0.8271

(a) Complete the table above giving the missing value of y to 4 decimal places.
(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 3 decimal places.
(3)

(c) Find

$$\int x^2 4^x \, dx$$

(5)

(d) Using your answer from part (c), find the area of region R, giving your answer to 3 significant figures.
(2)

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14. The curve C_1 has parametric equations

$$x = t^2 - 1, \quad y = t^3 - t \qquad\qquad t \in \mathbb{R}$$

The line l is the normal to C_1 at the point where $t = 2$

(a) Show that an equation of l is

$$4x + 11y - 78 = 0$$

(5)

The curve C_2 has parametric equations

$$x = 12.5 + a\cos t, \quad y = 15 + a\sin t \qquad\qquad 0 \leqslant t < 2\pi$$

where a is a constant.

(b) Find the range of values of a for which the curve C_2 does not cross or touch the line l.

(5)

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Question 14 continued

Question 14 continued

(Total 10 marks)

TOTAL FOR PAPER: 125 MARKS

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Q14